EFFECT OF FILTRATION FLOW ON THE EQUILIBRIUM SHAPE OF BODIES FORMED BY FORCED FREEZING

K. G. Kornev and G. I. Mukhamadullina

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The effect of filtration flow on the shape of a body formed by freezing around an isolated freezing column is investigated on the basis of the mathematical model proposed in [1].

We examine the problem of the limiting equilibrium shape of an ice–rock body formed around a freezing column. We use the formulation given in [1]. We assume that the filtration satisfies Darcy's law, the liquid is incompressible, and the thermophysical characteristics of the filtering medium are constant.

The mathematical model describing this process can be represented in the following dimensionless form:

$$\operatorname{div}\mathbf{V} = 0,\ \mathbf{V} = -\nabla p,\ z \in D;\ |\mathbf{V}| = 1,\ |z| \to \infty, \tag{1}$$

$$\mathrm{Pe}\mathbf{V}\nabla\theta = \Delta\theta,\ z \in D;\ \theta = 1,\ |z| \to \infty:\ \theta = 0,\ z \in \partial D_1, \tag{2}$$

$$\Delta\theta_1 = 0,\ z \in D_1;\ \frac{\partial\theta}{\partial n} = \frac{\partial\theta_1}{\partial n},\ \theta_1 = 0,\ z \in \partial D_1,$$
$$\lim_{r\to 0} r\frac{\partial\theta_1}{\partial n} = Q,\ z = 0, \tag{3}$$

where the dimensionless characteristics and the independent variables are as follows:

$$x = \frac{X}{l},\quad y = \frac{Y}{l},\quad \theta = \frac{t - t_p}{t_\infty - t_p},\ \theta_1 = \frac{\lambda_M (t_1 - t_p)}{\lambda_T (t_\infty - t_p)},$$

$$p = \frac{k\hat{p}}{lV_\infty},\ \mathrm{Pe} = \frac{K_c V_\infty l}{a_T},\quad \mathbf{V} = \frac{\hat{\mathbf{V}}}{V_\infty},\quad Q = \frac{q}{\lambda_T (t_\infty - t_p)}$$

where ℓ is the characteristic size of the body formed, and Pe is the Peclet number.

Here D is the region of filtration; D_1 is the region occupied by the solid body formed; ∂D_1 is the boundary of the body; t and t_1 are the temperature in the regions D and D_1, respectively; t_p is the freezing temperature of the flow; t_∞ and V_∞ are the temperature and velocity at infinity; and, q is the intensity of the freezing column.

We shall estimate the order of magnitude of the unknown maximum size of the body ℓ under the assumption that $\mathrm{Pe} \to \infty$. One can hope that the estimate obtained in this limiting case is an upper limit on the characteristic size of the body in the entire range of Pe numbers. From boundary-layer theory it is known that the heat flux from the side of the thawed zone is of the order of $O((t_\infty - t_p)\lambda_T\sqrt{\mathrm{Pe}}/\ell)$. The heat flux from the side of the frozen zone is $O(q/\ell)$. The condition that both quantities be of the same order of magnitude leads to the expression

$$l = \left(\frac{q}{\lambda_T (t_\infty - t_p)}\right)^2 \frac{a_T}{K_c V_\infty}. \tag{4}$$

In [1] the problem (1)-(3) is reduced to the following system of integral equations by methods of the theory of boundary-value problems for analytical functions:

N. G. Chebotarev Scientific-Research Institute of Mathematics and Mechanics, V. I. Ul'yanov–Lenin Kazan State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 61, No. 6, pp. 980-985, December, 1991.

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$$\pi = \int_{-1}^{1} \mu(\xi) K_0(P|t-\xi|) \exp(P(t-\xi))\, d\xi, \tag{5}$$

$$Q = 2\int_{-1}^{1} \mu(\xi)\, d\xi, \quad P = a\mathrm{Pe}, \tag{6}$$

$$F(\xi) = -\frac{1}{2\pi i}\int_{\partial\Omega_1}\left[\frac{\eta'(\xi)}{\eta(\tau)-\eta(\xi)} - \frac{1}{\tau-\xi}\right] F(\tau)\, d\tau + \ln G^*(\xi), \tag{7}$$

$$\ln a\chi^- = \frac{1}{2\pi i}\int_{\partial\Omega_1}\frac{F(\eta_{-1}(\xi))}{\xi-\omega}\, d\xi, \quad \omega\in\Omega_1, \tag{8}$$

$$\ln a\chi^+ = \frac{1}{2\pi i}\int_{\partial\Omega_1}\frac{F(\xi)}{\xi-\omega}\, d\xi, \quad \omega\notin\Omega_1, \tag{9}$$

$$z = \int_0^{\sigma}\frac{i\exp(i\tau)}{\chi^+(\exp(i\tau))}\, d\tau + \int_0^1\frac{d\tau}{\chi^-(\tau)}, \tag{10}$$

where

$$\xi = \exp(i\sigma), \quad \eta(\xi) = \exp(i\alpha(\sigma)),$$
$$\alpha(\sigma) = \frac{4\pi a}{Q}\int_0^{\sigma} G(\tau)\sin\tau\, d\tau, \tag{11}$$
$$G(\tau) = \begin{cases} +\mu(\cos\tau), & 0\leqslant\tau<\pi, \\ -\mu(\cos\tau), & \pi\leqslant\tau\leqslant 2\pi, \end{cases}$$

where $\eta_{-1}(\xi)$ is the inverse function to $\eta(\xi)$; Ω_1, $\partial\Omega_1$ are a circle of unit radius and its circumference, respectively; 4a is the length of the plate in the plane of the complex potential.

The number P is the physical parameter on which the solution of the system of equations (5)-(10) depends. It actually characterizes the effect of the filtration flow on the heat emission of the body and is an integral characteristic of heat transfer; this is evident from Eqs. (5) and (6). The shape of the body, determined from Eqs. (7)-(10), depends functionally on the heat-flux distribution on the plate (11). Thus analysis of the effect of filtration flow on the limiting equilibrium shape of the body reduces to studying the dependence of the distribution of the heat flux to the plate on the number P.

The analytical solution of Eq. (5), obtained in [2, 3] in the form of series in Mathieu and Airy functions, is inconvenient for further numerical calculations, so that this equation was solved numerically. For this, on the basis of the well-known theorems of [4] the solution of this equation was represented in the form

$$\mu(\xi) = \frac{\exp(P\xi)\mu_*(\xi)}{\sqrt{1-\xi^2}}$$

and then the equation was solved for $\mu_*(\xi)$ by the method of collocations.

For known $\mu(\xi)$ the determination of the shape of the body reduces to solving a system of linear equations. For this the function $F(\sigma)$ can be represented, by virtue of its analytic properties [5], in the form

$$F(\sigma) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k\cos k\sigma + ib_k\sin k\sigma.$$

The expansion of Eq. (7) in a Fourier series leads to a system of linear equations for the Fourier coefficients of the function $F(\sigma)$.

The number of nodes in the method of collocations and the number of coefficients in the Fourier series was determined in the course of the computational experiment.

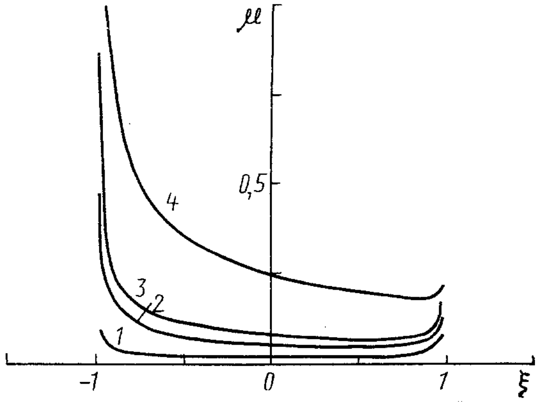

Fig. 1. Distribution of the heat-flux density: 1) P = 0.01; 2) 0.5; 3) 1; 4) 10.

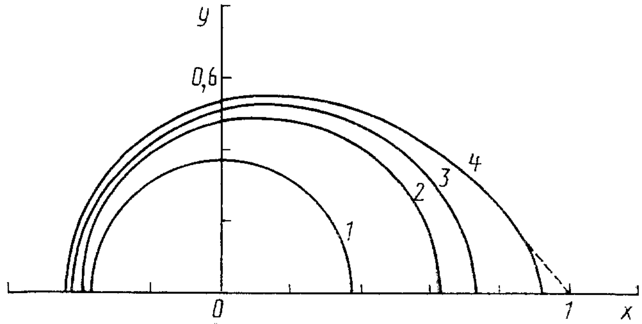

Fig. 2. Shape of the ice–rock body: 1) P = 0.01; 2) 0.5; 3) 1; 4) 10.

The results of the numerical calculations are presented in Figs. 1 and 2.

Figure 1 shows the distribution of heat fluxes to the plate, corresponding to an ice–rock body in the plane of the complex potential of the flow. For $P \leq 0.01$ the heat-flux density is distributed practically symmetrically and its value is the same as that calculated using the asymptotic formula [2, 6]. For this reason, the ice–rock body has a nearly circular shape (Fig. 2). Indeed, in this case the formula (11) assumes the form

$$\alpha(\sigma) = \sigma,$$

whence it is obvious that for small values of P the solution of the integral equation (7) is identically equal to 1.

Finally we obtain

$$z = \int_0^\sigma \frac{dt^+}{\chi^+} = a \exp(i\sigma), \quad \sigma \in [0, 2\pi].$$

Thus, for $P \leq 0.01$ the ice–rock body has a circular shape with radius a.

As one can see from Fig. 1, the heat-flux distribution becomes asymmetric as P increases, i.e., convective removal comes into play. As a result the ice–rock body assumes an ovoid shape.

In the region $P \geq 10$ the heat-flux density, calculated from the asyptotic formulas [7], is identical everywhere, with the exception of the vicinity of the back edge, to the heat-flux density obtained numerically. For this reason, the shape of the ice–rock body determined by the numerical method (solid lines) is different from the approximate solution obtained by Maksimov [8] (dashed line) only near the back edge (Fig. 2).

Thus for $P \leq 0.01$ the shape of the ice–rock body can be modeled by a circle of radius a. For $P \geq 10$ the shape of the bodyis described well by the equation obtained in [8] for large Pe numbers. For all other values of the parameter P the shape of the ice–rock body can be determined numerically using the algorithm described above.

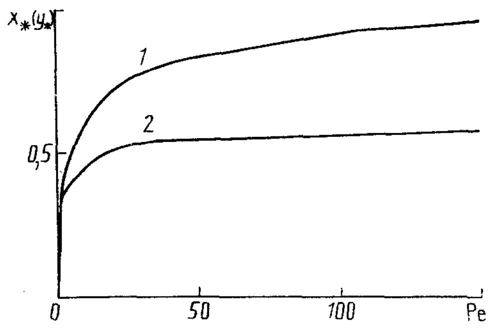

Fig. 3

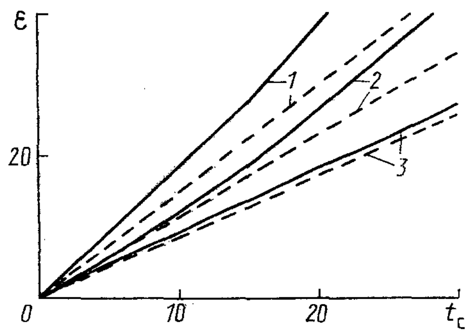

Fig. 4

Fig. 3. Maximum dimensions x_* (1) and y_* (2) of the ice–rock body versus the Peclet number Pe.

Fig. 4. Transverse size of the ice–rock body versus the freezing temperature: 1) V_∞ = 0.5 m/day; 2) 1; 3) 2; ε, cm; t_c, °C.

In practice, graphs of the maximum dimensions of the ice–rock body x_* and y_* versus the Peclet number Pe, which are presented in Fig. 3, can be used to estimate efficiently the required intensity of the freezing column.

We shall compare the solutions obtained with the experimental data of Prozorov [9], i.e., we shall study the case when the limiting ice–rock body was formed around a freezing column of radius r, on which a negative temperature t_c is maintained. We assume that $r/\ell \ll 1$. Since t_p = 0°C, the dimensionless temperature on the surface of the freezing column Γ can be represented as

$$\theta_1 = \frac{\lambda_M t_c}{\lambda_T t_\infty}.$$

On the other hand, the dimensionless temperature on Γ is equal to the real part of the complex thermal potential

$$\theta_1 = \frac{Q}{2\pi} \ln \rho,$$

where ρ is the radius of the circle into which the contour of the column is transformed by a conformal mapping of the unknown region D_1 on a circle of unit radius. Using the properties of conformal mappings we obtain, to within $O(r^2/\ell^2)$,

$$\rho = \frac{r}{l} \left. \frac{dt^{+}}{dz} \right|_{\zeta=0}.$$

Therefore,

$$\frac{\lambda_M t_c}{\lambda_T t_\infty} = \frac{Q}{2\pi} \ln \rho. \tag{12}$$

Substituting the equality Pe = Q^2, which follows from the formula (4), into Eq. (12), we obtain a formula for the freezing temperature

$$t_c = \frac{\sqrt{\mathrm{Pe}}}{2\pi} \frac{\lambda_T}{\lambda_M} t_\infty \ln \rho. \tag{13}$$

Figure 4 shows plots of the transverse size ε of an ice–rock body as a function of the freezing temperature t_c, calculated from Eq. (13), for different filtration rates V_∞ = 0.5, 1, and 2 m/day (solid lines). The dashed curves were obtained experimentally [9]. The thermophysical parameters in the calculation using Eq. (13) were taken from [9].

As one can see, the theoretical results agree satisfactorily with the experimental results. Here it should also be kept in mind that in [9] the experimental curves were obtained over a finite freezing time, while the theoretical limiting equilibrium state appears over an infinitely long time. For this reason, the theoretically computed temperature of the freezing column is always higher than the experimental temperature.

NOTATION

a, thermal diffusivity; λ, thermal conductivity; k, filtration coefficient; ℓ, characteristic size in the frozen zone; V, velocity; p, pressure; t, temperature; r, radius of the freezing column; Γ, surface of the freezing column; n, outer normal to ∂D_1; X and Y, Cartesian coordinates; ε, maximum transverse size of the ice—rock body. The dimensionless parameters, variables, and functions are: x, y, Cartesian coordinates; $z = x + iy$, complex variable of the physical plane; K_C, ratio of the volume heat capacities of water and soil; Pe, Peclet number; $\mu(\xi)$, heat-flux density; θ and θ_1, temperatures in the regions D and D_1, respectively; Q, intensity of the freezing column; $K_0(\xi)$, Bessel function. The indices are: T, thawed zone; M, frozen zone; c, surface of the freezing column; p, surface of the phase transition; ∞, value at infinity; *, maximum size of the ice—rock body; and, $\wedge$ designates dimensional variables.

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STUDY OF THE CHARACTERISTICS OF HEAT AND MASS TRANSFER IN A GAS—SOLID BODY SYSTEM AT LOW TEMPERATURES

N. V. Atapina, D. N. Garipogly,
A. S. Drobyshev, V. A. Kuz'min,
and S. L. Maksimov

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An experimental apparatus — a universal vacuum spectrophotometer — is described. It is used for studying the kinetic characteristics of gas—solid phase transitions at low temperatures as well as the spectral reflectances of typical optical surfaces coated with layers of cryocondensates of different gases. A method is proposed for determining the indices of refraction of the cryocondensates, the growth rates of the condensates, and the spectral reflectances.

Heat and mass transfer in cryogenic—vacuum systems in the presence of gas—solid phase transitions, while obeying general laws, nonetheless exhibits a number of important peculiarities which require special study. We are referring primarily to the presence of heat and mass fluxes which are determined by the transformation of gas into a solid phase and which can be determined in the systems studied. In addition, under certain conditions, namely, high vacuum and low temperatures, the radiation component makes an important contribution to the overall heat transfer. However, the appearance of layers of cryogenically deposited gases on the heat-transfer surfaces can significantly affect the reflectances of the surfaces and the parameters of the radiation heat transfer as a whole. Since there are no thoroughly developed methods for calculating heat and mass fluxes under these conditions, it is necessary to perform experimental investigations in this direction. It should be noted that interest in such investigations has increased substantially in the last 20 years. They are, however, of a fragmentary and particularly applied character. Here, first, we call attention to [1-3],

S. M. Kirov Kazakh State University, Alma-Ata.

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